LEARNING AND COMPETITION: A LEARNING CURVE ANALYSIS USEFUL FROM MARKETING TO TERRORIST ATTACKS

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Abstract – *Any doing involves learning. In order to analyze operations, learning curves have been used for decades. A rigorous mathematical reasoning for learning curves leading to exponential rather than power law behavior is relatively new. Up to now the learning of the acting party has been considered, only. This is perfectly okay for ordinary production where the material cannot "learn." However, in service-like operations the other side is learning too. Including both learnings will lead to two (coupled) learning curves. By fitting existing success data it is not only possible to predict the future. It is even possible to analyze the learning of the competitor. Data from the Afghanistan war are used to analyze the learning of the victims and the aggressors.*

Keywords – Analysis, Learning curves, Marketing effectiveness, War casualties, Red Queen

INTRODUCTION

"Rapitio mater sapientia est" is an old Latin proverb. Freely translated as "practice leads to improvement." In business learning curves describe the improvement due to repetition of similar tasks. Though they are used for decades in production like tasks, there is hardly any rigorous description how they are derived. Classically they take the form

$$cost = c_0 t^{-\alpha} \tag{1}$$

where $0 < \alpha < 1$ denotes an exponent giving the "speed" of learning and t is the time of trying. Besides fitting some real learning curves quite well, (1) must be wrong for general reasons: Cost does not tend to be zero if you are trying long enough. Eq. (1) is most likely derived from a random walk approach (for an overview see e.g. [1]). It is, e.g. a model of "learning" in competing biological systems. However, such "learning" is done by chance rather than analyzing and thinking. Therefore the random walk approach is fine for non-thinking structures such as animals or plants. Furthermore random walk can be performed forever leading to a possible infinite distance from the origin. This explains the incorrect $t \to \infty$ limit in (1).

It was quite recently [2] that a rigorous learning curve of the form

$$cost = (c_0 - c_\infty) \cdot e^{-t/\tau} + c_\infty \tag{2}$$

has been derived. There c_0 is the initial cost, c_∞ the final cost and τ a typical time period for learning. In production the engineer typically learns (consciously, not be chance) while the construction material stays "dumb." However, there are approaches where two parties learn. Typically in marketing people are fighting against competition. Both sides learn. Surprisingly, nobody ever considered such set-up. Up to our knowledge we are the first to derive learning curves where two sides are learning (cf. next chapter). The result is a formula with two exponential functions rather than one like in (2). Analyzing real success data will lead not only to the time scale of the own side. It will also reveal the learning of the competitor.

The idea to consider two learning parties came from a very recent article in The Economist [3]. There a paper of Neil Johnson [4] has been reviewed. The cumulative attack data were fit by a "learning curve" like given in (1). The fit was quite fine. Especially in [3] the question has been raised why the fit was so good. No interactive two party learning had been included. Taking one of the data sets from [4] (the one published in

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[3]) and applying our two party learning we result in a perfect fit. Furthermore we can identify the learning speeds of the two parties.

The rest of the paper is organized as follows. In the next chapter we will explain our two party learning model. Then we will apply our model to the data of [3] or [4]. In our conclusion we will comment on further applications, especially in competitor analysis and marketing.

DERIVING COUPLED LEARNING CURVES

The general approach to learning curves can be found in [2]. Denoting the quantity where the learning occurs by T (e.g. pieces produced or victims per attack) we have

$$\Delta T \ \propto T_\infty - T \tag{3}$$

where the index ∞ denotes the quantity after perfect learning (= best possible outcome). The learning is proportional to the distance from the optimum. Essentially it means, e.g. that finding one of ten errors will take you have the time of finding one of five errors. Eq. (3) will lead to a differential equation with a solution in the form of (2). With two learning parties the other side will also learn. Their variable is T_∞. So we have

$$\frac{dT}{dn} = c_1 \left(T_{\infty n} - T\right) \tag{4}$$

$$\frac{dT_{\infty n}}{dn} = c_2 \left(T_\infty - T_{\infty n}\right) \tag{5}$$

where the index n denotes the nth try. The c's are the inverse time scales of party one or two, respectively. Eqs. (4) and (5) are two coupled linear differential equations. Their solution is straight forward. Eq. (5) can even be solved separately. Separation of variables and integration of (5) yields:

$$T_{\infty n} = \ T_\infty + \left(T_{\infty 0} - T_\infty\right) e^{-c_2(n-1)} \tag{6}$$

Eq. (4) is now easily solved by inserting the solution (6) in (4). This differential equation is solved by using an ansatz of two exponential equations in the form

$$T = \ T_\infty + A \cdot e^{-c_1(n-1)} + B \cdot e^{-c_2(n-1)} \tag{7}$$

By using the initial condition $T(1) = T_1$ and differentiating (5) with respect to n, inserting it in (4), and inserting (7) in it, the constants A and B are easily determined:

$$A = \ T_1 - \frac{c_1}{c_1 - c_2} T_{\infty 0} + \frac{c_2}{c_1 - c_2} T_\infty \quad and \quad B = \frac{c_1}{c_1 - c_2} \left(T_{\infty 0} - T_\infty\right) \tag{8}$$

Please note that the limit $c_1 \rightarrow c_2$ does not cause trouble, though it does considered in (8) alone. Inserting (8) in (7) and taking the limit $c_1 \rightarrow c_2$ is perfectly fine. It leads to one party learning as it should. When seeking a numerical fit of existing data (cf. next chapter), it is much smarter to use (7) alone rather than inserting (8) in (7) beforehand. Else most algorithms will fail.

Eq. (7) (together with (8)) is the general learning curve where two parties learn. The variable T may be understood as e.g. revenue and n the number of the marketing campaign. The c_1 is then the speed of learning how to optimize marketing. c_2 is the speed how the competitors learn to counter-attack the marketing campaigns. In the next chapter we will consider terrorist attacks and the corresponding protective action.

ANALYZING ATTACK DATA FROM TERRORISTS

To start with, this analysis is not taken to understand a particular terrorist attack and its corresponding counter action much better. For that reason we must have much better data of which the origin and its quality is perfectly well-known. The main reason we are picking on this example is that this data has been examined recently ([3], [4]). Furthermore it is a good example to explain the principle usage of (7) and (8).

In [4] (and [3]) the attacks of terrorists on allied forces in Farah (western Afghanistan) had been examined. These data has been fitted by (7). The result is:

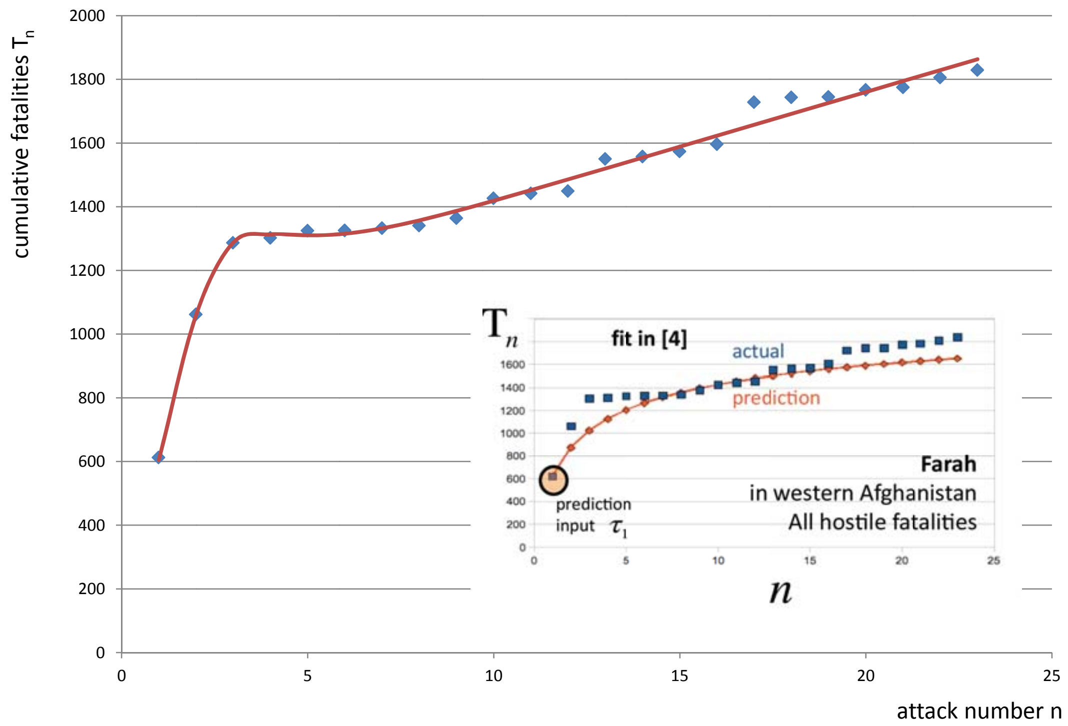

FIGURE 1
Fatalities due to terrorist attacks and fit in [4]

The diamonds denote the actual data and the curve is our fit with the integral of (7). The integration from 1 to n over (7) is necessary, because cumulative data are considered here. Integration of (7) yields:

$$T = T_\infty(n-1) - \frac{A}{c_1}\cdot\left(e^{-c_1(n-1)} - 1\right) - \frac{B}{c_2}\cdot\left(e^{-c_2(n-1)} - 1\right) \quad (9)$$

The nonlinear fit is tedious but straight forward. From it all constants can be determined:

$$A = -2467 \qquad B = 4391 \qquad c_1 = 0.7923 \qquad c_2 = 1.040 \quad (10)$$

As one sees both sides are learning, though with different speeds. Though the aggressors (c_1) learn slower in (8) than the defenders (c_2), it can also be vice versa. (A and B are also changing roles then) It cannot be decided by our fit for principle reasons. The fit in our diagram is much “nicer” than the one in [3] or [4]. However, this is not the main point hear. The use of more fit parameters than in [4] easily explains our high accuracy. (As stated above we are not sure whether we misunderstood the data from [4]. Even then it is easy to fit the points with four parameters) Our point is that we used a newly developed theory of two party learning. Fitting real data reveals the speed of learning of each side without knowing anything about the underlying operations. This is a very mighty tool, because the defender has no good knowledge of the aggressor. In some sense one can spy on the other side without using agents or satellites. This is especially important in business, where spying on the other side is not only difficult but strictly illegal. Our way is simple, inexpensive, and perfectly legal.

Another way to use our results is to predict either the future or to analyze different scenarios. In our example above one may consider the situation where the learning of either side is faster or slower. The result is displayed in figure 2:

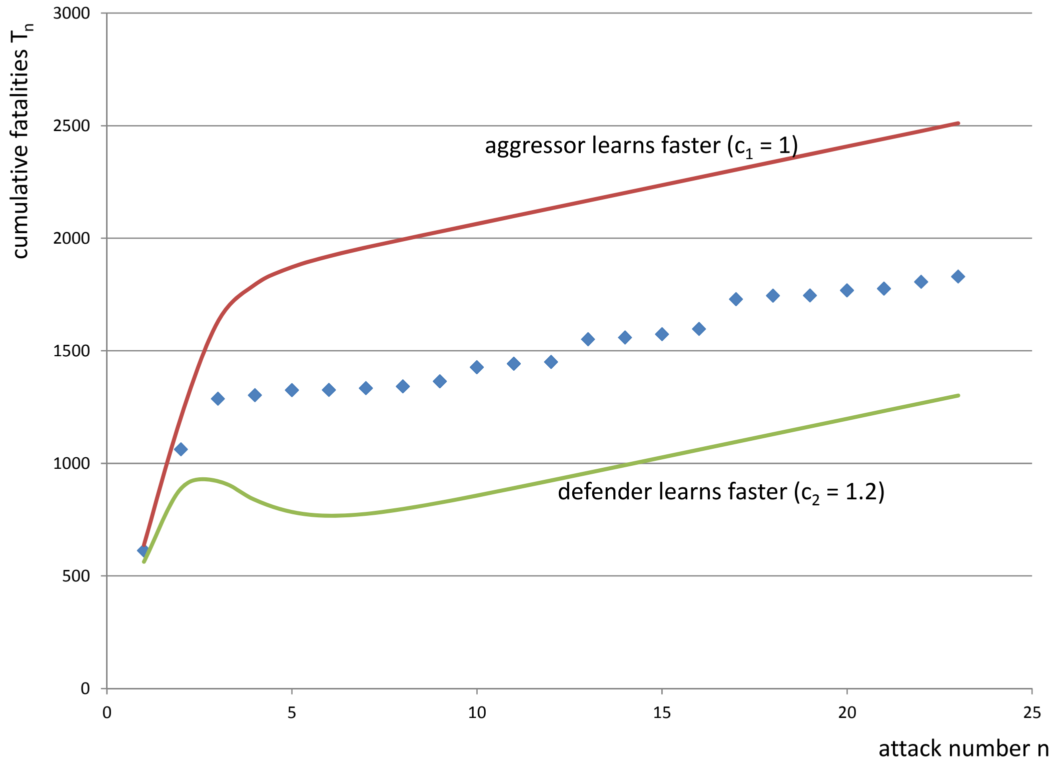

FIGURE 2
Change in fatalities due to different learning speeds

Had the defender learnt faster, total fatalities were significantly lower (lower curve of figure 2). Unfortunately the opposite is also true. Had the aggressor learnt faster (upper curve of figure 2), the total number of fatalities were much higher. Please note that in the equilibrium state ($n \rightarrow \infty$, here reached around n = 10 already) we have always the same constant fatality rate (here around 34 fatalities per attack).

FURTHER APPICATIONS AND CONCLUSIONS

In most kind of enterprises there are many competing sides. It is always possible to consider one's own enterprise as one party and all the competitors combined as the second party. Then we have two parties. None of them is normally perfect in the beginning. Both are learning by eliminating their mistakes or errors. We have shown that there are two coupled exponential learning curves. The result is a quite simple formula (7).

The result is applicable and useful as long as there are competing sides. So we cover most situations where human beings interact. It reaches from mating over competing sports teams to marketing campaigns of competing companies [5]. Of course, the latter one is of main interest here. Typically companies start marketing campaigns in order gain market share. And the competitors will start counter-campaigns. Just by observing these changes in market share and fitting the results by (7) one will get a detailed and quantitative inside of one's own success and even more importantly of the success of the competitors.

Of course, quantitative analysis is far from being new in marketing. Originally coming from psychology, conjoint analyses conquered marketing around 1970 already. (As an exemplary reference see [6] and references therein) However, in all such stupendous work there is a common problem. How to get reasonable data? Measuring the desired quantities like market share, revenue, or profit is simple and quite accurate.

Finding out what part is due to a certain marketing campaign is next to impossible. Other quantities like brand recognition surely have an impact on profit. But what percentage of profit increase is due to higher brand recognition? Our suggested analysis cannot answer these questions either. However, such questions are unimportant within our analysis. We analyze relative rather than absolute data. The *shape* of the curve in figure 1 is important rather than its absolute values. For our analysis it is not important which percentage of market share increase is due to marketing. As long as there is a certain percentage it is good enough. Also quantities like brand recognition are perfectly fine to analyze how one's own company learn to improve marketing and how fast the competitors are learning. That does not mean that any kind of data are equally fine. In the beginning one should list all measurable data. Then one should look which of them are reasonable if taken as T and n in the differential equations (4) and (5). (Slight changes in (4) and (5) may also be useful) The details are beyond this short publication.

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